Taking care of field curvature in visual optical instruments

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ABSTRACT

Instruments intended for use with the human eye almost always suffer from field curvature, because nearly all the components in them are of positive power. This paper will look at some standard eyepiece designs, and also one of the designs undertaken by Ancient and Modern Optics, comparing the calculated dioptre variation across the field with the subjective impressions of the user. Finally a design methodology for visual systems will be proposed which takes into account the asphericity, as well as the curvature, of the field presented to the eye, and the ability of the eye preferentially to select the best of the two astigmatic fields presented to it.

Keywords: Field curvature, astigmatism, OSLO, eyepiece.

1. INTRODUCTION

Astigmatism and field curvature are the two Seidel aberrations which are the principal subject of this paper. The formulae for the Seidel sums, which give a first-order approximation to the actual aberrations, using the notation of Hopkins **[1]** and Welford **[2]** are:

$$S_{III} = -\Sigma\bar{A}^2 h\Delta(u/n)$$

for astigmatism and

$$S_{IV} = -H^2\Sigma c\Delta(1/n)$$

for field curvature. For a thin single element positive lens of power K (=1/F) of refractive index n these become:

$$S_{III} = H^2 K$$

assuming the lens is at the stop, and

$$S_{IV} = \frac{H^2 K}{n}$$

which is independent of stop position. The curvature of the image surface of a system free of astigmatism is known as the Petzval curvature, and for an image formed in air the vertex radius of curvature of the image is:

$$R_P = \frac{H^2}{S_{IV}} = n\,F$$

If the astigmatism is nonzero, then the rays in the tangential section will intersect on a surface with vertex radius:

$$R_T = \frac{H^2}{(3S_{III} + S_{IV})} = n\,F\,/\,(3n+1)$$

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and, in the sagittal section, on a surface with radius:

$$R_S = \frac{H^2}{(S_{III} + S_{IV})} = n\,F\,/\,(n+1)$$

The surfaces and their centres of curvature are shown in figure 1 for a typical glass lens.

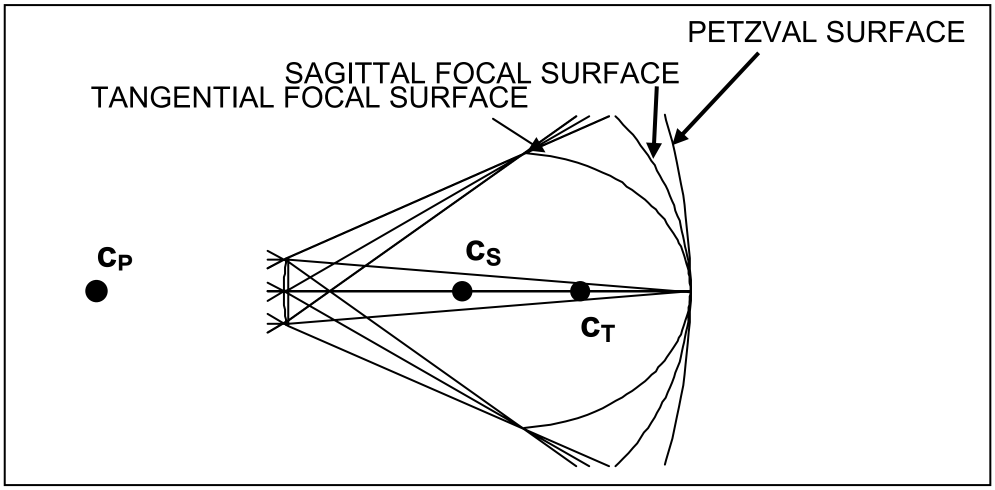
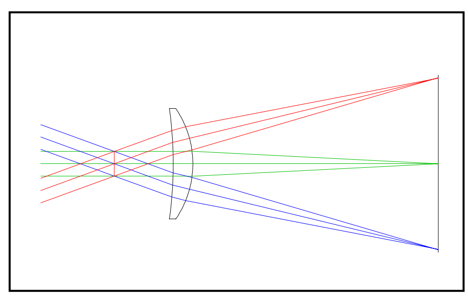


Fig. 1. Diagram showing the shapes of the astigmatic surfaces for a thin single element of refractive index 1.5 with the stop at the lens. Note that the centres of curvature only apply to the paraxial region, since the actual image focal surfaces are parabolic.

There are two significant factors which apply to any optical system to be used in conjunction with the human eye. First, almost without exception, the sum of the powers of the individual components will be positive. In practice this means that to keep the entire field of view within the accommodation range of the eye, it will be necessary to bring the axial focus in from infinity, to ensure that the image at the margins of the field of view will not go beyond infinity. In other words, rays from any point in the field of view reaching the eye must be either parallel or diverging.

The second point is that the stop of the system will be at the eye, which is outside the optical system. This means that the stop shift effects will come into play. In particular, this means that any spherical aberration and coma will also have an effect on the astigmatism. This design technique is the basis of the landscape photographic lens used in the XIXth century, a single element bent steeply towards the stop which induced astigmatism which compensated for the field curvature (see figure 2).

Fig. 2. A "landscape" f/10 wide angle photographic lens with the stop in front. There is an alternative design, with similar performance, which has the stop behind the lens. This is more practical in terms of keeping dirt out.



2. ASTIGMATISM AND FIELD CURVATURE IN SOME CLASSIC DESIGNS

The eyepieces and magnifiers shown here, mainly obtained from the OSLO public lens database, have all been scaled to the same focal length to enable comparisons to be made between the astigmatism and field curvature of each design, traced from a plane object at infinity in the eye space. The OSLO command **plf chr** is used to generate the plots of the astigmatic focal surfaces for all three wavelengths.

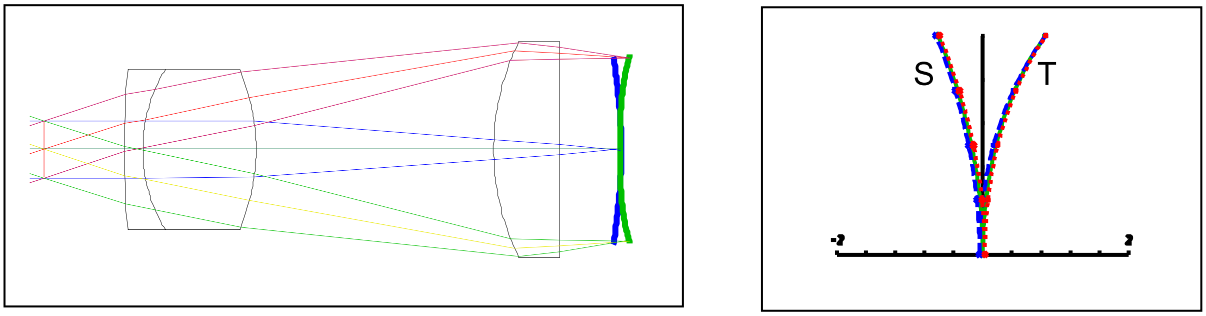


Fig. 3. A simple Kellner eyepiece with a 36° field of view, showing very little field curvature, but with astigmatic focal surfaces at equal distances on either side of the nominal focal plane.

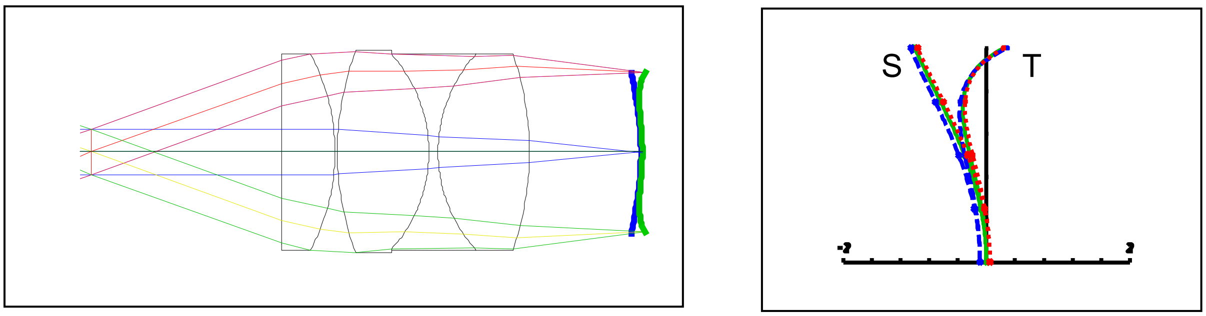


Fig. 4. An orthoscopic eyepiece with a 40° field of view, showing very little astigmatism in the axial region, but with increasing astigmatism at the edge of the field.

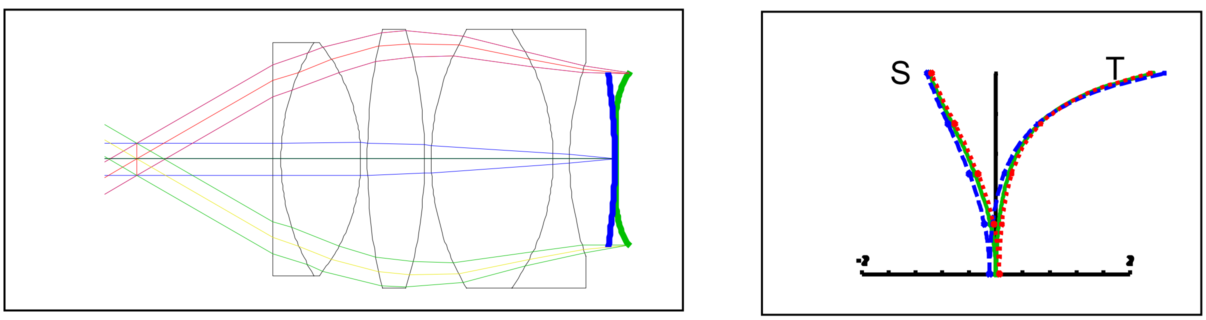


Fig. 5. An Erfle type eyepiece with a 60° field of view with a relatively flat sagittal image surface.

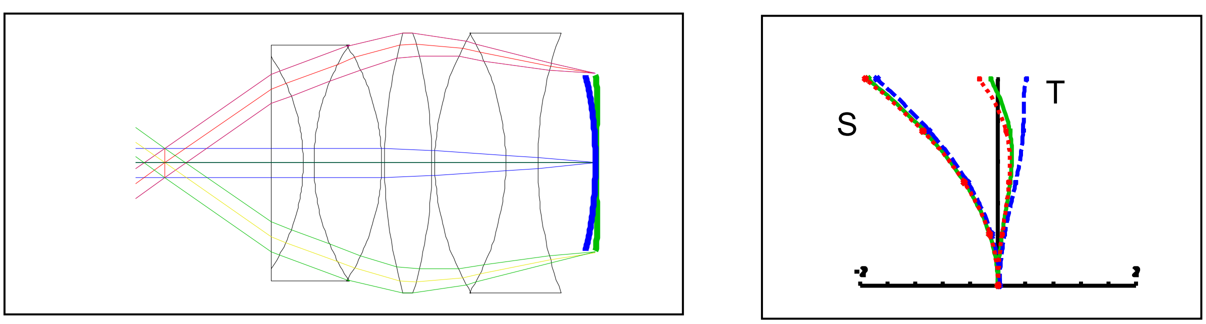


Fig. 6. A wider angle Erfle eyepiece with a 70° field of view, with a nearly flat tangential field.

The method by which the focal surfaces in the lens diagrams were prepared is worth commenting on here. A merit function controlling aberrations of rays only in the meridional section in one case, and only skew rays in the other, was used to find the optimum shape of the two image surfaces, with curvature, conic constant and fourth- and perhaps sixth-order aspheric coefficients as variables.

Note that the last three of these eyepieces, which have large fields of view, have one, but not both, of the image surfaces relatively flat. A possible explanation of why this has been seen as a desirable design goal is the ability of the human eye presented with two images, one in focus and one out of focus, to selectively adapt to the best. This unexpected result was the basis of the success of the bifocal contact lens, which was patented by Pilkington Perkin Elmer Ltd **[3]**. This lens covered part of the pupil with a prescription for reading, and the other part with a prescription for distance vision. We might suggest that the mechanism which made this invention work is the same mechanism by which the eye can reject an image - either in the sagittal section or in the tangential section - which it cannot accommodate.

Of course these analyses - tracing parallel ray bundles in reverse from the eye space to a fictitiously curved, and perhaps aspheric "image" - do not represent the actual situation. In each of the two principal sections, the object (perhaps a self-luminous screen) is imaged on to a virtual surface a finite distance in front of the eye, which is arbitrarily curved and often aspheric. The author has showed that optimisation using the correct method of tracing can radically alter the design of the final system.

3. DESIGN OF A MODULAR IMAGING SYSTEM

The customer requirement shown in figure 7 **[4]** was for a modular viewing system to relay the image of a flat high resolution CRT with a 21 inch screen diagonal. This was to be relayed over varying distances up to 2.6 metres long, and the image presented to the eye with a diagonal field of view of 68°. Because of the diameter restriction on the relay optics, there was no opportunity for components of negative power to offset the inevitable field curvature arising from the combination of so many positive components.

The simplicity of the final design made it possible for the prototype to be constructed mostly from components obtained from a catalogue supplier. The one exception was the final objective, a standard photographic objective for a 35mm camera obtained secondhand from a photographic shop.

The components, starting from the eyepiece and numbered as in the diagrams, were as follows:

1. 32 mm Erfle eyepiece
2. Achromatic doublet F = 400 mm, diameter 75 mm.
3. Stop 1
4. Achromatic doublet F = 400 mm, diameter 75 mm.
5. Planoconvex single element, F = 175 mm, diameter 50 mm
6. Achromatic doublet F = 400 mm, diameter 75 mm.
7. Stop 2
8. Achromatic doublet F = 400 mm, diameter 75 mm.
9. Planoconvex single element, F = 200 mm, diameter 50 mm
10. Achromatic doublet F = 400 mm, diameter 75 mm.
11. Stop 3
12. Achromatic doublet F = 400 mm, diameter 75 mm.
13. Planoconvex single element, F = 150 mm, diameter 50 mm
14. Planoconvex single element, F = 150 mm, diameter 50 mm
15. Objective, F = 48.6 mm, f/6, 40° diagonal field of view.

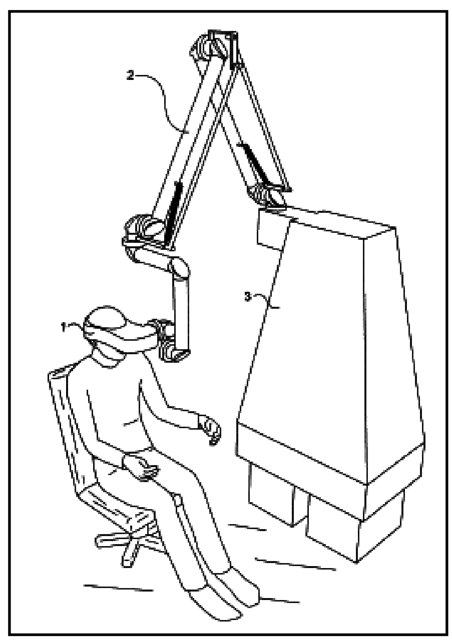


Fig. 7. Schematic layout of the virtual reality headset.

Note that the three relays are not identical. The three field lenses, 5, 9 and 13+14, have different focal lengths, since the effect of the pupil aberrations means that the different amounts of longitudinal pupil aberration need to be accommodated in the different relay stages.

The customer for this project wished to assess experimentally the appearance of an image projected through a relay train of varying length. Since the design was modular it made it possible for the subjective effects of field curvature to be assessed in the three situations where one relay, two relays, and three relays were used. The design of the three relay version is illustrated in figures 8 to 11.

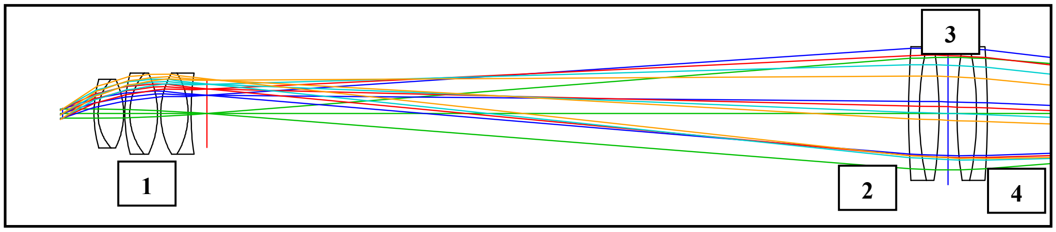


Fig. 8. The first section of the virtual reality viewing system demonstrator, consisting of an Erfle type eyepiece, and the first relay lens which consists of a pair of identical cemented doublets with stop between.

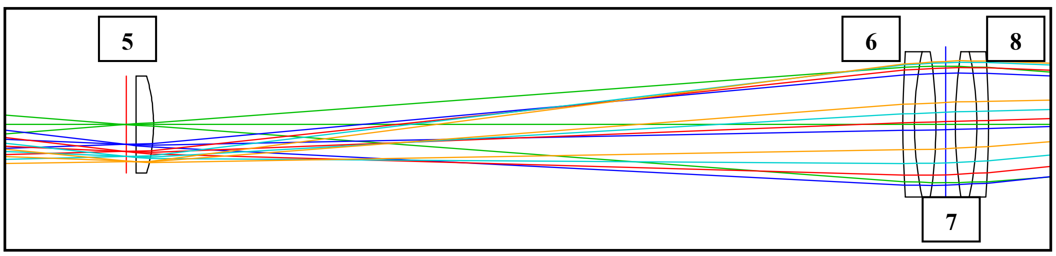


Fig. 9. The second intermediate image, field lens and second relay.

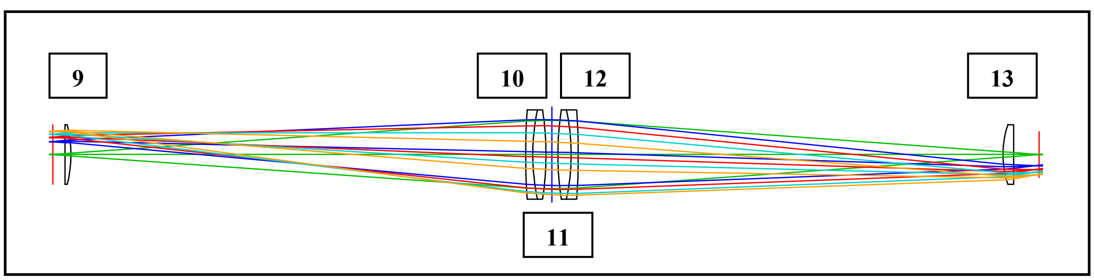
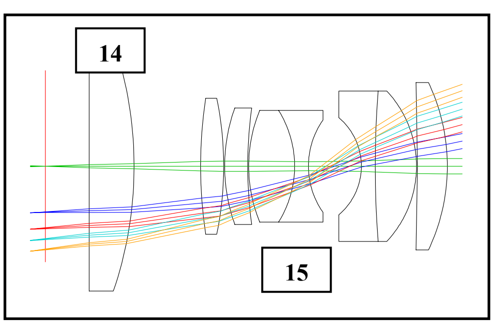


Fig. 10. The third intermediate image, field lens, third relay, and the first of the two field lenses at the last intermediate image.



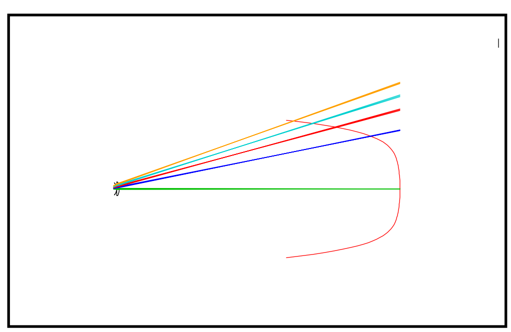


Fig. 11. The final field lens and projection objective. Fig. 12. The best fit focal surface at the distal end of the periscope.

Figure 12 shows the shape and position of the optimum focal surface for collimating beams projected through the eyepiece. This is obtained using the normal optimisation merit function, with only the curvature, conic constant, and fourth order aspheric coefficients of the image as variables. An alternative method of analysis, and one which preserves the distinction between the two focal surfaces, is obtained by regarding the system as an afocal telescope, and plotting the dioptre variations in focus in the image space for an object at infinity at the eyepiece. The results for the entire system are shown in figure 13, which shows clearly the increase in field curvature as relay stages are added.

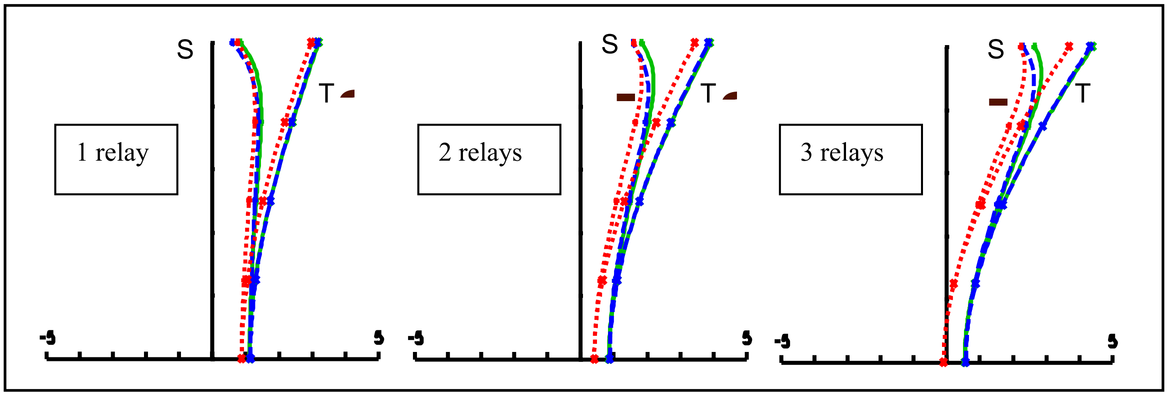


Fig. 13. Field surfaces for an object at infinity, with curvatures measured in dioptres, for the three options. The horizontal scale is in dioptres.

The subjective impression obtained by the author when viewing the three different versions of the instrument were that field curvature was imperceptible for the system with one relay, only just perceptible for the system with two relays, and was becoming very uncomfortable for the system with three relays.

4. DESIGN OF A COMPACT UNITY MAGNIFICATION TELESCOPE

Another optical system in which field curvature plays an important part has been the subject of a recent project undertaken by Ancient and Modern Optics. The requirement was for a head-mounted unity magnification telescope with well controlled distortion, covering a field of view of 40° with a 16 mm diameter exit pupil.

To meet the customer's requirements of erect orientation of the image, the most economical way of achieving this was by the provision of two intermediate images, as in the old-fashioned naval telescope. The optical system will then consist of an objective, a relay and an eyepiece. Now if the objective and the eyepiece are identical, the relay itself may consist of a pair of identical objectives of arbitrary focal length. This means that the whole system can become symmetrical about a plane, which acts as the aperture stop of the entire system. This symmetry implies that only half the system, that is to say a telescope of arbitrary magnification, consisting of an objective and eyepiece, needs to be designed. The complete system will then consist of a pair of these telescopes, placed with their objectives face-to-face about the central stop of the system.

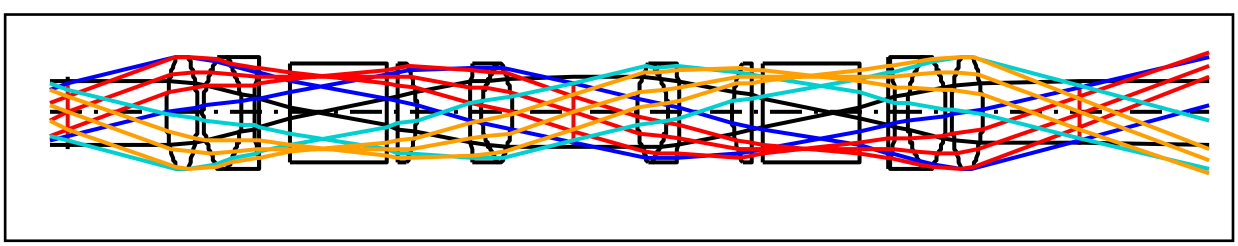


Fig. 14. The unit magnification telescope, which consists of a pair of telescopes mounted with the two objectives face-to-face, so that their entrance pupils coincide at the central stop.

The design is illustrated in figure 14. The two individual telescopes have magnification of less than unity (x0.88), which helps to keep the overall length to the desired value. This symmetry ensures that the magnification is unity, and that the odd order aberrations, that is to say distortion, coma and transverse chromatic aberration, are by definition zero.

As can be seen from the diagram, there are only components of positive power in this design. As may be expected, therefore, field curvature and astigmatism are the major limiting factors in the performance. As before, afocal analysis of the field aberrations, shown in figure 15, confirms that the field curvature in the sagittal section to be 6.5 dioptres, and more in the tangential section. So we have to ask the question, is this acceptable?

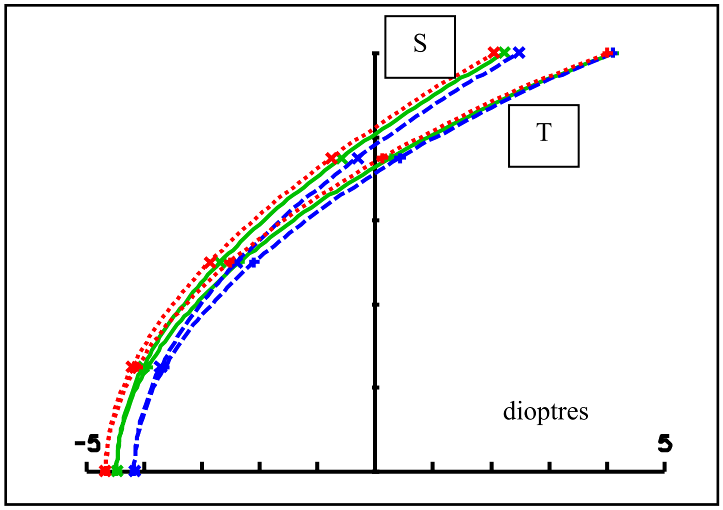


Fig. 15. Field curvature and astigmatism for the unity magnification telescope. For the purposes of assessment, the assumption has been made that the axial image is presented at -4.5 dioptres (that is an image distance of 222 mm).

The two textbooks which gives the best description of the human eye and its relationship to optical instruments are both by David Atchison and the late George Smith**.** The first of these references **[5]** gives a table which shows the results of tests on the amplitude of accommodation as a function of age carried out among a sample population of civilians (principally civil servants) in Australia in 1986. Briefly, the result is that the mean amplitude of accommodation is 9.5 dioptres at 20 years of age, 7.5 dioptres at 30, and 5 dioptres at 40.

The corresponding results in the second reference **[6]** indicate broadly similar values for one method of measurement, but for the stigmatoscopic method of measurement the results are lower. It is believed that this method of measurement is less relevant in the present application, so these results can be discounted.

What these results imply is that field curvature in this instrument over the field between the axis and a total field of 32° (from -4.5 to 0 dioptres) will not be noticeable since the anticipated users are under 40 years of age. Outside this circle (with a positive dioptre setting) it will not be possible for normal users of any age to accommodate the image. However peripheral cues of brightness and in particular of movement will still be perceptible. As this instrument is intended for head mounting, peripheral image quality does not have the same degree of importance as would be the case in a fixed instrument.

The last two figures show graphically the shapes of the optimised images (mean of sagittal and tangential) formed by the instrument. Figure 16 illustrates graphically the shape of the paraboloid representing the image surface, which extends towards infinity as the field angle increases. At a field angle of 32° (total), the image is at infinity. Beyond that it is no longer possible to accommodate the image. This is illustrated in figure 17, showing the image field behind the user's head, and outside the range of normal accommodation. In this case, because the instrument is entirely symmetrical, there is no distinction between the tracing of rays from a flat object to a curved image and vice versa.

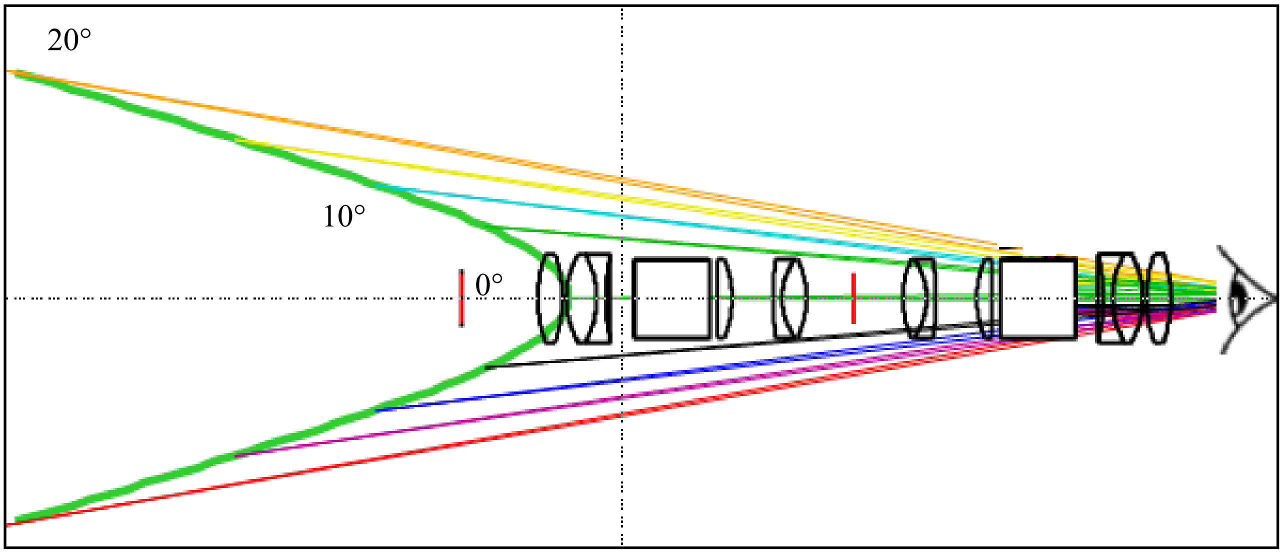


Fig. 16. Optimised focal surface shape (compromise between sagittal and tangential) for field angles up to 20° total in the unity magnification telescope.

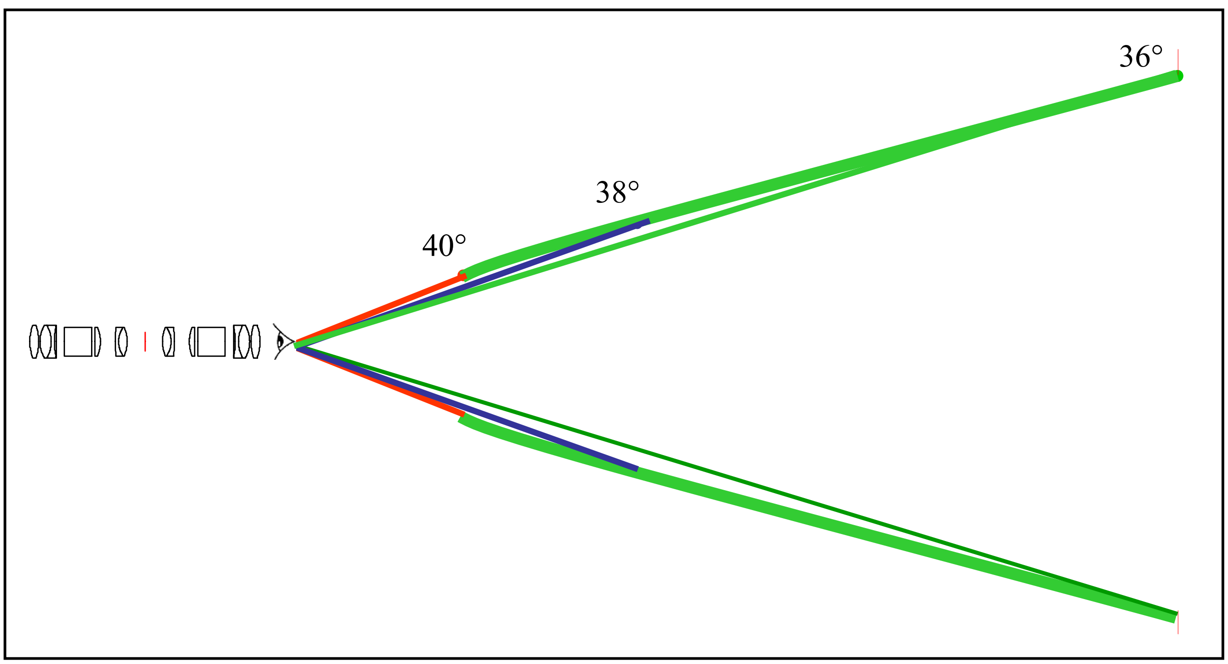


Fig. 17. Optimised focal surface for field angles from 36° to 40°.

5. CONCLUSION

The assessment of a visual instrument by tracing in reverse from a flat object at infinity towards a fictitiously curved image, is not an acceptable approximation if the field angle is significant. Nor, if astigmatism is present, can the ability of the eye to reject one of the astigmatic fields (either sagittal or tangential) be discounted. The suggested design procedure for a visual instrument is therefore: (1) Tracing in the reverse direction from the image to the object, with the "object" flat and at infinity, assess the preliminary design and find the least curved of the two "image" fields. (2) If the field curvature of this lies outside acceptable limits, optimise this field - if necessary by at the expense of the other. (3) Set the focus so that the no part of the preferred field goes beyond infinity. (4) Fixing the lens parameters, set the image as flat (or whatever is the actual shape), and then optimise the object shape to give a best fit surface to the best of the astigmatic fields. Alternatively, if software permits, the lens may be treated as a zoom lens, with the object distance as a parameter which changes from one field point to the next. (5) Finalise the optimisation with rays which represent these actual rays seen by the eye, if necessary allowing the object shape and position to change as well. (6) Check the dioptre range of the least curved field in the final design against the accommodation amplitude of the user. For an eyepiece or magnifier with fewer surfaces than the 10-surface limit of OSLO-EDU™ **[7]** this procedure can be demonstrated using the OPIC optimisation routine.

ACKNOWLEDGEMENTS

I would like to acknowledge the contributions made to this paper by my colleagues in Ancient and Modern Optics, Alison Fairhurst, and Leslie Shadrake. Some assistance has also been provided by Konrad Anders of the University of Applied Sciences in Jena, who is currently working with us as an intern.

Finally I would like to dedicate this paper to the memory of Dr George Smith, a long-term friend, who passed away in January of this year after a brief illness.

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* The OSLO optimisation routine OPIC and its documentation may be downloaded from the www.amoptics.com website. Other optimisation routines which can be used only with OSLO Standard and OSLO Premium will be available shortly. OSLO is a registered trademark of Lambda Research Corporation.